# Analyses of Magnetohydrodynamic Propulsion with Seawater for Underwater Vehicles

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### Introduction

THE magnetohydrodynamic (MHD) propulsion of marine vehicles using seawater's conducting characteristics has been a subject of technical speculation and study for some time.<sup>1,2</sup> The concept did not appear to hold much promise until the advent of the superconducting magnets. With these magnets, the power requirement for excitation is virtually absent, and the weight penalty for the ferromagnetic core is reduced. Also, much stronger magnetic fields than those previously attainable can be realized. Its technical possibility was assessed by sea runs in the past,<sup>2</sup> in which the vehicle ran slowly with low efficiency. It was then concluded that superconducting magnets are very essential. However, even with superconducting magnets, the weight problem remains because of the need for structural materials to maintain the integrity of the magnets. Whether an MHD propelled vessel is economically viable is a question awaiting further investigation. However, in certain naval applications where the importance of reducing acoustic signature outweighs other considerations, MHD propulsion offers the desired quietness due to its reduced mechanical moving parts.

### Theoretical Analyses

Similar to the way liquid metal is pumped,<sup>3</sup> in MHD thrusters, the electric current and magnetic field are arranged to be orthogonal to each other in the channel to provide the Lorentz  $(j \times B)$  force. Seawater is pumped in an active section of the channel and passed through a nozzle. An MHD thruster without the converging nozzle is also possible in which the main thrust is the pressure thrust. Two configurations of submerged vehicles with duct-type thrusters are shown in Figs. 1a and 1b. The annular thruster gives higher efficiency because the friction area per unit volume of fluid in the channel is minimized. However, the scheme with separate rectangular channels provides the maneuverability by individually controlling the thrust of each channel.

# Analysis of the MHD Pump

Simple rectangular MHD channels have been analyzed before. The analysis showed that the mechanical power imparted on the seawater and the electric power needed in the channel are

$$P_{\rm w} = \frac{1 - \eta_{\rm ind}}{\eta_{\rm ind}} \, \sigma B^2 U_{\rm in}^2 V_{\rm ch} \tag{1}$$

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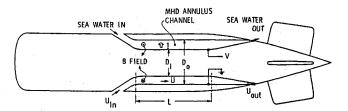


Fig. 1a Submersible with an annular MHD channel.

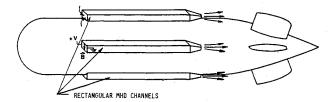


Fig. 1b Submersible with rectangular MHD channels.

$$P_e = VI = \frac{1 - \eta_{\rm ind}}{\eta_{\rm ind}} \sigma B V U_{\rm in} W L \tag{2}$$

where B is the magnetic field;  $U_{\rm in}$  the velocity in the channel;  $\sigma$  the conductivity of seawater;  $V_{\rm ch}$  the active volume of the channel defined as  $V_{\rm ch} = DWL$ , where L, W, and D are the electrode length, width, and gap distance, respectively;  $\eta_{\rm ind}$  the field induction efficiency related to the back emf as  $\eta_{\rm ind} = BU_{\rm in}D/V$ ; V the applied voltage, and I the net current across the channel. The electrical efficiency is defined as, from Eqs. (1) and (2),

$$\eta_e = \frac{P_w}{P_e} = \frac{BU_{\rm in}D}{V} \tag{3}$$

This is identical to the field induction efficiency  $\eta_{\text{ind}}$ . This, however, does not mean one can extract the most mechanical power from the MHD channel at  $\eta_e$  (or  $\eta_{\text{ind}}$ ) = 1. As one can see from Eq. (2),  $P_e$  approaches zero as  $\eta_e$  approaches unity. From Eqs. (1) and (3), the mechanical power at given channel dimensions and electrical potential becomes

$$P_w = (1 - \eta_e)\eta_e \sigma V^2 \frac{V_{\text{ch}}}{D^2} \tag{4}$$

 $P_w$  can be optimized by taking the derivative of Eq. (4) with respect to  $\eta_e$  and determining the value of  $\eta_e$  that satisfies a zero derivative. Hence,  $P_w$  is optimized when  $\eta_e = 0.5$ . This means 50% of the electrical power will be consumed in Joule heating. This is a condition that must be taken into consideration in the thruster design.

### **Dual-Control-Volume Analysis**

Previous analyses of seawater MHD thrusters have ignored the friction of fluid in the channel, and the pressures at the entrance and exit  $(p_{in} \text{ and } p_{ex})$  were assumed the same as the ambient pressure.<sup>4</sup> These assumptions are not necessarily valid. In the current analysis, two separate control volumes are considered to take into account the friction in the channel. Pressures at the inlet and outlet of the channel must be computed. The first control volume is the seawater volume enclosed by an MHD channel. If the ratio of the nozzle exit area to the MHD channel area is s, the mass conservation yields,

$$U_{\rm ex} = U_{\rm in}/s \tag{5}$$

where  $U_{\rm ex}$  is the exit velocity of the nozzle. The momentum conservation yields

$$\rho U_{\text{in}} A_{\text{in}} (U_{\text{ex}} - U_{\text{in}}) = IBD - \left( f \frac{L}{D_H} \frac{\rho U_{\text{in}}^2}{2} \right) A_{\text{in}}$$
$$- \Delta p_N A_{\text{in}} + (p_{\text{in}} A_{\text{in}} - p_{\text{ex}} A_{\text{ex}})$$
 (6)

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The second term on the right-hand side (RHS) of Eq. (6) accounts for the friction loss in the straight channel and the third term accounts for the nozzle loss.  $D_H$  is the equivalent hydraulic diameter; f the friction coefficient, which is a function of the Reynold's number; and  $A_{\rm in}$  and  $A_{\rm ex}$  the entrance and exit areas of the channel, respectively. The pressure loss across the nozzle  $\Delta p_N$  is proportional to  $U_{\rm in}^2$  and is obtained from the nozzle equations. The last term on the RHS of Eq. (6) is the net pressure force from both the inlet and outlet of the channel.

The thrust produced by an MHD channel can be written from the momentum principles as

$$T = \dot{m}(U_{\rm ex} - U_{\rm in}) + (p_{\rm ex}A_{\rm ex} - p_{\rm in}A_{\rm in}) + p_{\rm amb}(A_{\rm in} - A_{\rm ex})$$
 (7)

where the first term on the RHS is the momentum thrust and the combination of the second and third terms is the pressure thrust;  $p_{amb}$  is the ambient pressure of the vehicle and is a depth-dependent quantity. From Eq. (6), Eq. (7) can be written as

$$T = \Delta p_{\rm ch} A_{\rm in} + p_{\rm amb} (A_{\rm in} - A_{\rm ex}) \tag{8}$$

where  $\Delta p_{\rm ch}$  is the pressure rise across the entrance and exit of the MHD channel. It is defined as

$$\Delta p_{\rm ch} A_{\rm in} = (p_{\rm ex} - p_{\rm in}) A_{\rm in} = IBD$$

$$-\left(f \frac{L}{D_H} \frac{\rho U_{\rm in}^2}{2}\right) A_{\rm in} - \Delta p_N A_{\rm in}$$
(9)

The relationship between the total thrust of all of the MHD channels and the vehicle velocity can be obtained from the second control volume, which encloses the outer surface of the vehicle. The skin friction of a vehicle is proportional to the square of its velocity and must be balanced by the total thrust provided by the MHD thrusters. Therefore,

$$C_D A_{\text{surf}} \left( \frac{1}{2} \rho V_s^2 \right) = N_{\text{ch}} T \tag{10}$$

where  $N_{\rm ch}$  is the total number of MHD channels,  $A_{\rm surf}$  the total vehicle surface area, and  $C_D$  the drag coefficient on the vehicle surface. The thrust in Eq. (7) can also be reduced to

$$T = \dot{m}(U_{\rm ex} - U_{\rm in}) - (p_{\rm in} - p_{\rm amb})(A_{\rm in} - A_{\rm ex}) + \Delta p_{\rm ch} A_{\rm ex}$$
 (11)

and the term  $(p_{in} - p_{ex})$  can be approximated to an entrance pressure defect as

$$p_{\rm in} - p_{\rm amb} = \frac{1}{2} \rho (V_s^2 - U_{\rm in}^2) \tag{12}$$

From Eqs. (11) and (12), Eq. (10) becomes

$$C_D A_{\text{surf}}(V_2 \rho V_s^2) = N_{\text{ch}}[\dot{m}(U_{\text{ex}} - U_{\text{in}}) - \frac{1}{2} \rho (V_s^2 - U_{\text{in}}^2)(A_{\text{in}} - A_{\text{ex}}) + \Delta p_{\text{ch}} A_{\text{ex}}]$$
(13)

### **Solution Procedures**

In the previous discussion, the pressures at the entrance and exit of an MHD channel were treated as unknowns. The relation between them was formulated in Eq. (9). If dimensions of the vehicle and MHD channel are given, the applied voltage that gives the optimal electric efficiency can be determined for given B and  $U_{\rm in}$  by letting  $\eta_{\rm ind}=0.5$ . Then,  $\Delta p_{\rm ch}$  can be calculated from Eq. (9) for a given s. Equation (13) becomes a quadratic equation of  $V_s$ , which can be solved. However, there exists a unique s<1 for a given  $U_{\rm in}$  to satisfy the condition of Eq. (8). It is also noted that Eq. (8) involves the ambient pressure. For a given depth, iterative procedures must be used to solve the value of s.

Here, it is convenient to define the three terms on the RHS of Eq. (13) as the momentum thrust, pressure thrust 1, and

pressure thrust 2, respectively. It is noted that, when s=1, both the momentum thrust and pressure thrust 1 are zero, yielding Eq. (13) the same as Eq. (8). That means in the present study s=1 is always a solution. However, the solutions (s<1 or s=1) may not be physically valid. It depends on whether the resulting thrust power will violate the imposed 50% electric efficiency.

### Results and Discussion

Calculations were based on two classes of underwater vehicles. The first class (class 1) was vehicles with dimensions close to MK48 torpedoes. The diameter and length were chosen to be 0.533 and 6.1 m, respectively. The second class (class 2) was large submersibles like submarines, with diameter and length being 9.8 and 83 m, respectively. Four rectangular channels

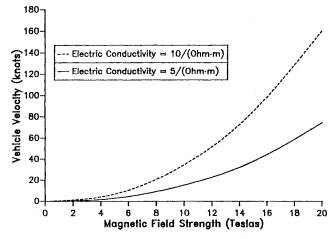


Fig. 2 Velocity vs magnetic field for class 1 vehicles for B = 20 T.

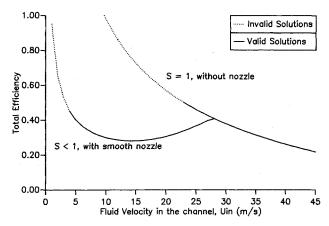


Fig. 3a Total efficiency of class 1 vehicles for B = 20 T.

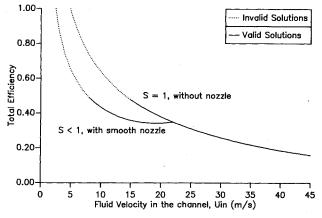


Fig. 3b Total efficiency of class 2 vehicles for B = 5 T.

were attached to the vehicle 90 deg apart from one another, and the channel lengths were chosen to be two-thirds of the vehicle lengths. For class 1 vehicles, the electrode width and gap distance were 0.3 and 0.1 m, respectively. For class 2 vehicles, the electrode width and gap distance were 3 and 1 m, respectively. The vehicles were assumed to operate at an ambient pressure corresponding to a depth of 30 m below the surface.

Figure 2 shows the dependence of class 1 vehicle velocity on the magnetic field and the seawater conductivity. The vehicle velocity increases proportionally with  $\sigma B^2$ . The solid line represents normal seawater conditions. For typical torpedo speeds (50-70 kt), the magnetic field needs to be 15-20 T. These are enormously large fields. When the conductivity of seawater in the channel is increased, shown as the dashed line, the velocity is increased linearly. Figures 3a and 3b show the efficiency performances of class 1 (B = 20 T) and class 2 (B = 5 T) vehicles vs the fluid velocity in the channel, respectively. For both classes, the vehicle velocity increases with channel flow, as a result of increased MHD pumping. As discussed earlier, there exist two satisfactory solutions for  $V_s$ at a  $U_{in}$ . One solution corresponds to s = 1. This solution, shown as the upper curves in Figs. 3a and 3b is not valid when  $\eta_t > 0.5$  since it contradicts the imposed condition of electric efficiency. The other solution corresponds to s < 1. It suggests that the existence of a nozzle would create a higher  $U_{ex}$ , which generates a momentum thrust. This is shown as the lower curves in the figures. Valid solution normally exists for s = 1 at higher velocity. In this situation, the vehicle is propelled by the pressure thrust.

Reasonable submarine velocity can be attained with a magnetic field of about 5 T, Fig. 3b. For a vehicle having 35-kt velocity, the total efficiencies are about 0.35 and 0.45 for s < 1 and s = 1, respectively. This is largely due to the large size of the MHD channels. The electric power will likely come from nuclear. For example, a class 2 vehicle with a speed of 36 kt needs 66 MW to power its MHD channels. Assuming the cycle efficiency of the nuclear propulsion plant is 33%, the required reactor thermal power would be 200 MW. This does not include any other service power needed for the vehicle. Nevertheless, it is a power level achievable in the current nuclear submarine technologies.

## Acknowledgment

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